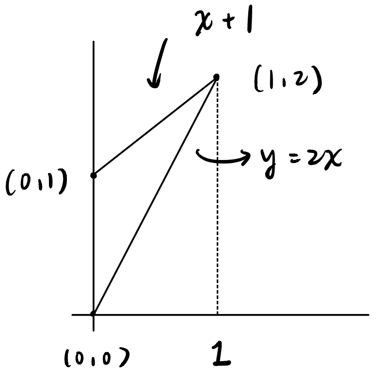
Class Exercise 10

1. Evaluate the line integral
$$\oint_C x^2y\,dx + x^3\,dy\ ,$$
where C is the boundary of the triangle at $(0,0),(0,1)$ and $(1,2)$ in anticlockwise direction.



$M = x^2y,\quad N = x^3$

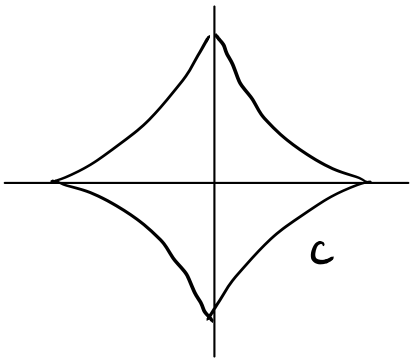
$\frac{\partial M}{\partial y} = x^2,\quad \frac{\partial N}{\partial x} = 3x^2$

$\int_0^1 \int_{2x}^{x+1} 3x^2 - x^2 \quad dy\,dx$

$= \int_0^1 \int_{2x}^{x+1} 2x^2\,dy\,dx$

$= \cdots$

2. Find the area of the region enclosed by the hypocycloid $\cos^3 t\mathbf{i} + \sin^3 t\mathbf{j},\ t \in [0, 2\pi]$.



using Area $= \frac{1}{2}\oint_C -y\,dx + x\,dy$

$\Rightarrow \frac{1}{2}\int_0^{2\pi} -\sin^3 t \cdot 3\cos^2 t \cdot (-\sin t)\ dt$

$+ \frac{1}{2}\int_0^{2\pi} \cos^3 t \cdot 3\sin^2 t\cos t\ dt$

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